A Brazilian Wunderkind Who Calms Chaos

Artur Avila’s solutions to ubiquitous problems in chaos theory have earned him Brazil’s first Fields Medal.

By Thomas Lin and Erica Klarreich

It was pouring rain on a chilly spring day, and Artur Avila was marooned at the University of Paris Jussieu campus, minus the jacket he had misplaced before boarding a red-eye from Chicago. “Let’s wait,” said the Brazilian mathematician in a sleep-deprived drawl, his snug black T-shirt revealing the approximate physique of a sturdy World Cup midfielder. “I don’t want to get sick.” In everyday matters, Avila steers clear of complications and risk. Afraid his mind will veer from road signs and oncoming traffic to “unimodal maps” and “quasi-periodic Schrödinger operators,” he doesn’t drive or bike. “There are too many cars in Paris,” he said. “I’m fearful of some crazy bus killing me.”

Soon the conversation turned to a different kind of worry for Avila — that public reminders of Brazil’s apparent lack of intellectual achievement will discourage students there from pursuing careers in pure math and science research. In the lead-up to this summer’s World Cup competition, popular news websites and TV shows like “Good Morning Brazil” parroted the question: How has the world’s seventh-largest economy managed to score five World Cup titles but zero Nobel Prizes? (The British biologist Peter Medawar’s tenuous connection to Brazil — born there but raised in his mother’s native England — merits at best an asterisk.) Even Argentina, that bitter soccer rival with a population one-fifth the size of Brazil’s, boasts five Nobel laureates.

To Avila, the criticism stings. “It’s not good for the self-image of Brazil,” he said.

Even then, in May, the native son of Rio de Janeiro had a secret weapon, a compelling argument that Brazil belongs among elite math nations like the United States, France and Russia. But he could tell no one — until today. The International Mathematical Union has made Avila the first Brazilian recipient of the Fields Medal, awarding the 35-year-old what many consider the equivalent of a Nobel Prize in mathematics for his “profound contributions to dynamical systems theory” that “have changed the face of the field,” according to the prize selection committee.

“He’s one of the very best analysts in the world,” said Jean-Christophe Yoccoz, a renowned Collège de France mathematician and 1994 Fields medalist. Of the many talented postdoctoral researchers Yoccoz has advised, he said, “Artur is in a class by himself.” Most mathematicians focus on a narrow subfield and have a low success rate, Yoccoz explained, but Avila “attacks many important problems and solves many of them.”

His work “cannot be reduced to ‘one big theorem’ as Artur has so many deep results in several different topics,” said Marcelo Viana, who worked with Avila to solve a long-standing problem about
the chaotic behavior of billiard balls. The two proved a formula that predicts which side of the table a ball is most likely to hit next — and which side it will likely hit after a thousand bounces, or a million, all with the same margin of error. By contrast, Viana observed, if you try to predict the weather, “you’ll get very good predictions for tomorrow, not very good predictions for the day after, and completely lousy predictions for 15 days from now.”

Months before today’s announcement on the IMU website, the Brazilian dynamicist Welington de Melo predicted that his former doctoral student would win math’s highest honor. “It’s going to be extremely important to Brazil,” he said. “We never before got such a high prize. It is especially important because Artur was a student in Brazil all the time.”

Math on the Beach

Two things Avila fears more than erratic buses are PowerPoint slides and income tax forms. The pressure to perfect a plenary talk for the thousands attending the 2010 math congress in Hyderabad, India, induced in him a kind of mental paralysis, he said. After giving a lecture at the California Institute of Technology in 2008, he declined an honorarium of more than $2,000 just to avoid the paperwork.

“I would get fired pretty fast from most jobs,” he said, adding that he sleeps well past noon and is “not good at managing time.”

But in mathematics, Avila has a reputation for diving headfirst into unfamiliar waters and rapidly solving a raft of ambitious open questions. His colleagues describe his working style as highly collaborative and freakishly fast and Avila himself as having a clear-minded intuition for simplifying deep complications.

“He has high geometric vision,” said Raphael Krikorian, an Armenian-French dynamicist at Pierre and Marie Curie University in Paris. “He tells you what you should look at, what you should do. Then, of course, you have to work.”

Now a globe-trotting dual citizen of Brazil and France, Avila spends half the year in Paris as a research director at CNRS, France’s largest state-run science organization, and the other half in Rio as a fellow at IMPA, Brazil’s national institute for pure and applied mathematics. (The Brazil-France connection is no coincidence; in the 1970s and 1980s, top young French mathematicians like Étienne Ghys and Yoccoz fulfilled their mandatory military service with a civil-service alternative: conducting research at IMPA.)

In balmy Rio during the summer and winter months, Avila mulls over problems while lying in bed or wandering Leblon beach a block from his apartment. There, he has more time and freedom to think deeply about his work and to let ideas flow freely. “I don’t believe that I can just hit my head on the wall and the solution will appear,” he likes to say. He sometimes invites collaborators to Rio one at a time for what can only be described as an unconventional work experience.
spends about half of the year in Paris, where he prefers walking over other forms of transportation.

“The last time I was in Rio, I specifically got a hotel near the beach so I could work with him,” said Amie Wilkinson, a mathematician at the University of Chicago. After searching for Avila on a beach that was “packed shoulder-to-shoulder” with “oversexed cariocas” and returning to her hotel to try to call him, Wilkinson eventually found him “literally standing in the water,” she said. “We met and worked up to our knees in water. It was totally crazy.”

“If you work with Artur,” she said, “you have to get into a bathing suit.”

Avila was born to parents who could not envision their son growing up to become a pure mathematician — they had never heard of one — and wanted him to aim for a stable career as a bureaucrat. His father’s formal education growing up in the rural Amazon didn’t start until his teenage years, but by the time Artur was born, his father had become an accountant in a government reinsurance enterprise, able to provide a middle-class lifestyle in Rio for his family and buy math books for his quiet son, who early on was more interested in reading than imitating Pelé’s bicycle kick. When Avila was 6, his mother — who still files his tax returns — enrolled him at Colégio de São Bento, a conservative Catholic school known for its academics and for the gold-plated, 16th-century São Bento Monastery. Two years later his parents separated. As the years passed, Avila increasingly focused on mathematics to the exclusion of almost everything else — he often did poorly in other subjects and was expelled after the eighth grade for refusing to take mandatory religion exams. He said he “left the school completely unprepared for normal social interaction.”

Avila got his first taste of the wider mathematics community just before he was expelled in 1992 when Luiz Fabiano Pinheiro, a master teacher at São Bento affectionately known as “Fabiano,” encouraged the 13-year-old prodigy to participate in the junior division of the prestigious Mathematical Olympiad competition. Avila was excited by problems he had never encountered but felt woefully unprepared. “For the first time, I felt I couldn’t do anything,” he said. The next year, after Fabiano helped him transfer to a new school, Avila won top honors at the state level. Two years later, he took gold at the International Mathematical Olympiad in Toronto.
“The first time I met Artur, I knew that he would be pre-eminent,” Fabiano said in Portuguese as his ex-wife, Eliana Vianna, interpreted. “Artur was the best of all my students ever,” said the retired 72-year-old who taught for five decades.

Through the math competitions, Avila discovered IMPA, where Brazil held its Olympiad award ceremonies each year. There, he met prominent mathematicians like Carlos Gustavo Moreira and Nicolau Corção Saldanha, and while still technically in high school, he began studying graduate-level mathematics.

**Dynamical Systems**

In Brazil, Avila could relish mathematics without the career pressures he might have faced in the United States. “It was better for me to study at IMPA than if I were at Princeton or Harvard,” he said. “Growing up and being educated in Brazil was very positive for me.”

A major focus at IMPA is dynamical systems, the branch of mathematics that studies systems that evolve over time according to some set of rules — a collection of planets moving around a star, for example, or a billiard ball bouncing around a table, or a population of organisms that grows or declines over time.

One reason that many young mathematicians are drawn to dynamical systems, several researchers said, is that the relatively new subject, unlike the ancient field of number theory, doesn’t require a great deal of prior theoretical knowledge to begin solving problems. And dynamical systems are everywhere in math and nature. “It’s like a glue that connects many other subjects,” Krikorian said. Of the “two cultures of mathematics” described by the University of Cambridge mathematician and 1998 Fields medalist Timothy Gowers, there are theory-builders who create new mathematics and there are problem-solvers who analyze existing questions. Most dynamicists, said Yoccoz, including Avila and himself, are problem-solvers. “Both ways are necessary,” he said.

In the decades preceding Avila’s work, mathematicians made a profound discovery: To produce complex behavior, it isn’t necessary to start with complex rules. Even simple rules when repeated again and again sometimes produce chaos: random-seeming, unpredictable behavior in which tiny changes in the starting conditions can produce dramatically different outcomes. One of the first simple systems in which chaotic behavior was discovered is the so-called “logistic” model of population growth, which gives a precise formulation for how a population will change from year to year. Avila arrived on the scene just in time to write what could be considered the final chapter of this story.
The Logistic Equation

In the logistic equation for population growth, a rate $r$ between 1 and 4 controls the steepness of the parabola, reflecting how drastically the population responds to an abundance or shortage of resources. For values of $r$ between 1 and 3, the population size eventually settles down to some equilibrium value.

But when $r$ hits 3, something strange happens: Instead of a single equilibrium, there are suddenly two different population sizes that cycle back and forth in alternating years. Most starting populations gradually tend toward this cycle. Then, at approximately $r = 3.44949$, there's another change: The cycle splits into four different population values that cycle around over a period of four years. Around $r = 3.54409$, this cycle splits into a cycle of eight, and these “period-doubling bifurcations” keep happening, faster and faster, until, at approximately $r = 3.56995$ — a value that mathematicians call the “onset of chaos” — the system becomes highly unpredictable. Instead of settling down to an equilibrium or a cycle, the population swings wildly around from year to year, hitting every value between 0 and 1 with equal likelihood. The discovery, one of the earliest examples of chaos in a simple mathematical system, helped propel the study of chaos into a full-fledged branch of mathematics.

In nature, a small population often grows quickly because there is an abundance of resources; a larger population will grow more slowly or even decline as resources are stretched too thin. In 1838, the Belgian mathematician Pierre Verhulst captured this intuition in the logistic equation for population growth. The graph of the logistic equation is simply an upside-down parabola that rises quickly if the population is small but drops precipitously if the population is larger than the environment can sustain. As a population changes in time, it will move around on the parabola — a small population may become large the next year and a large one small.

Not all populations respond the same way, of course. The logistic equation encodes this diversity with a parameter $r$ between 1 and 4 that controls the steepness of the parabola: Higher values of $r$ correspond to populations that react in more abrupt, extreme ways to small changes. Populations with low values of $r$ will tend toward an equilibrium point or perhaps bounce around between a few values from year to year. But for certain values of $r$ larger than about 3.56995 — a value called the “onset of chaos” — the system becomes completely unpredictable.

“It’s just a parabola — something kids learn how to graph in school,” Wilkinson observed. “Yet that simple picture has this insanely rich stuff going on.”

Researchers had known for decades that beyond the onset of chaos, there were “islands of stability” — values of $r$ greater than 3.56995 for which the population would tend, for example, to a three-year cycle or a seven-year cycle. In the late 1990s, Mikhail Lyubich of Stony Brook University in Long Island elucidated what happens outside of these islands: For almost every other parameter beyond
the onset of chaos, the equation’s behavior is “stochastic,” exhibiting the unpredictable bouncing around that is the hallmark of chaos.

Lyubich had recently completed his analysis of the logistic equation when he traveled to IMPA in 1998. While there, he met Avila and instantly put the shy 19-year-old at ease. “As a student, I was scared of making mistakes,” Avila said. “He was very gentle and not scary at all.”

Lyubich, de Melo and Avila decided to try to extend Lyubich’s analysis of the behavior after the onset of chaos to a more general setting. In the mid-1970s, mathematicians discovered that the particular mix of cycles and chaos that the logistic equation exhibits seems to be a universal feature of every family of equations with the same basic shape as the upside-down parabola (called unimodal maps). Scientists also uncovered this same combination of cycles and chaos in a wide variety of systems in fluid dynamics, chemistry and other areas of science. Researchers struggled, though, to ground their observations in formal mathematics.

The three mathematicians investigated what happens to a family of unimodal maps after the onset of chaos. They distilled the problem down to a particular question, which Avila then solved. His construction, Lyubich wrote in a survey of Avila’s work in 2012, “is delicate and first looked too good to be true — but it worked, and it completed the argument.”

The proof showed that a wide class of unimodal families behaves just like the logistic family: After the onset of chaos, there are islands of stability, surrounded nearly entirely by parameters that give rise to stochastic behavior.

**Hammer and Nails**

In the spring and fall months, when he is based at CNRS in Paris, Avila is a mathematical free electron, moving from institute to institute in search of “attractive” problems. “Sometimes beauty is found in the mathematical statement and sometimes in the use of mathematical tools,” he said. “When they mix together in an unexpected way, then it’s something that I want to be working on.”

Many mathematicians are drawn to Avila because he “demystifies” complicated ideas, making them seem trivial, said Bassam Fayad, a CNRS colleague. “If you work with him, this experience changes your attitude toward mathematics. You kind of learn to do math without pain.”

Once, while wandering the streets of Paris with Avila, Jairo Bochi mentioned that he was trying to use spheres to construct a proof that he had been working on for two months. “People usually take a long time to understand your problem,” said Bochi, a mathematician from southern Brazil who shared an apartment with Avila when they studied at IMPA. “He immediately could see what I was saying and made a suggestion: ‘The spheres won’t work; try a compact cylinder.’” It solved the problem, and in 2006, they published their result.
Avila, right, frequently visits his collaborator and former postdoctoral adviser, Jean-Christophe Yoccoz, at Collège de France.

When jumping into a new field, Avila prefers to learn by talking to other people, rather than reading past papers. “Several times, I have gone aggressively to a new problem without having a lot of background in the area,” Avila said. “Maybe, since I don’t know what other people have been trying to do, I avoid some dead ends. Once I get a good result, then I’m more motivated to learn about the field, to understand what I’m proving.”

Central to the proof on unimodal maps was a powerful technique called renormalization that can sometimes convert one dynamical system into a new, related system by zooming in on a small part that behaves similarly to the system as a whole. Avila became a master of this technique. “He contributed in a deep way to understanding this phenomenon,” Lyubich said.

Renormalization was Avila’s hammer, and he started realizing that nails were lying around everywhere he looked. He carried out a series of further collaborations on unimodal dynamical systems, which, Lyubich wrote in the survey, “effectively closed up the field.” But that was just the beginning. Avila began diving into other areas of dynamical systems, using renormalization to solve one important problem after another.

“Part of Avila’s strength is that he is capable of working in all these different areas and, in a sense, unifying them,” Lyubich said. “He selects an area that looks interesting, finds the right fundamental problem to work on, then goes after it and is basically unstoppable.”

Avila has also studied the evolution of quantum states in physical systems governed by “quasi-periodic Schrödinger operators,” which are crude models for quasicrystals, structures that have more order than a liquid but less than a crystal. For these quasi-periodic Schrödinger operators, Avila — working with Svetlana Jitomirskaya of the University of California, Irvine, and David Damanik of Rice University in Houston — “just came along and cleaned up,” Wilkinson said. “He
answered a ton of questions about them.” One of these questions, concerning the energy states that electrons can take on in a particular model for a quasicrystal, had been known informally as the “ten martini” problem because it was so difficult that the late Polish mathematician Mark Kac had promised 10 martinis to anyone who could solve it. Avila has recently been extending this understanding to an entire family of quasi-periodic Schrödinger operators.

Avila has worked with Wilkinson and Sylvain Crovisier of the Université Paris-Sud in Orsay, France, to study a famous hypothesis of the 19th-century Austrian physicist Ludwig Boltzmann. Boltzmann proposed that a gas inside a box is “ergodic,” meaning that the gas atoms will rapidly travel through all possible arrangements, rather than, for example, hanging out for extended periods in particular regions of the box. In recent work, Avila, Crovisier and Wilkinson have shown that in the case of mathematical models whose behavior is at least moderately smooth, Boltzmann’s ergodic hypothesis is true, except possibly in the case of certain systems that are highly predictable, akin to a billiard ball bouncing around a table.

Although the vast majority of his papers have been collaborations, it has been years since Avila’s former doctoral adviser worked with him. “I think he’s too fast for me,” said de Melo, 67. “You have to work very hard to try to keep up with him. He would be happy to do almost everything, but I want to make sure I’m also contributing.”

**Brazilian Superstar**

Jet-lagged and stubble-faced after his trans-Atlantic flight in May, Avila looked more like a postdoc than a master dynamicist with a wide-ranging anthology of greatest hits. Having made important contributions to his field since he was 19 (and having earned his Ph.D. at 21), he has long defied expectations. When it comes to math, if not helter-skelter city traffic or tax filings, Avila’s shy anxieties have been replaced by a relaxed confidence and unwavering determination.

Fayad, who has known Avila since the Brazilian wunderkind first arrived in Paris in 2001, spoke of his friend’s growing drive and professionalism, whether he is attacking big math problems or researching weight-training techniques on the Internet. “He’s not very amateur in things,” Fayad said. “Suppose he wants to eat chocolate. He will become a professional eater of chocolate.”

For all his early accomplishments, Avila insists that he doesn’t set goals for himself, preferring to let his work unfold naturally. “Most of the time when I accomplish something, it’s not because I had a goal but because I was doing something that I wanted to do,” he said. “I just want to keep enjoying doing math.”

And he hopes his home country will share his enthusiasm. In addition to Avila’s Fields Medal win this year, Brazil is set to host the 2017 International Mathematical Olympiad and the 2018 International Congress of Mathematicians, where the next Fields medalists will be announced. “These four years are naturally going to be a good period to develop math in Brazil,” Avila said.

He hopes it is only the beginning of a transformative movement that will elevate assumptions about his country’s intellectual promise.

Last year, at a small bar in Paris that happened to be playing Brazilian music, Avila overheard a French mathematician tell his friends that Brazil “did not really have a math school.” When Avila objected, the other mathematician made it clear that he was aware of IMPA.

“I was a bit annoyed and insisted there was high-level math coming from there,” Avila said, to which the French mathematician replied, yes, there is that Brazilian superstar — Avila or something. “He
was expecting someone older.”