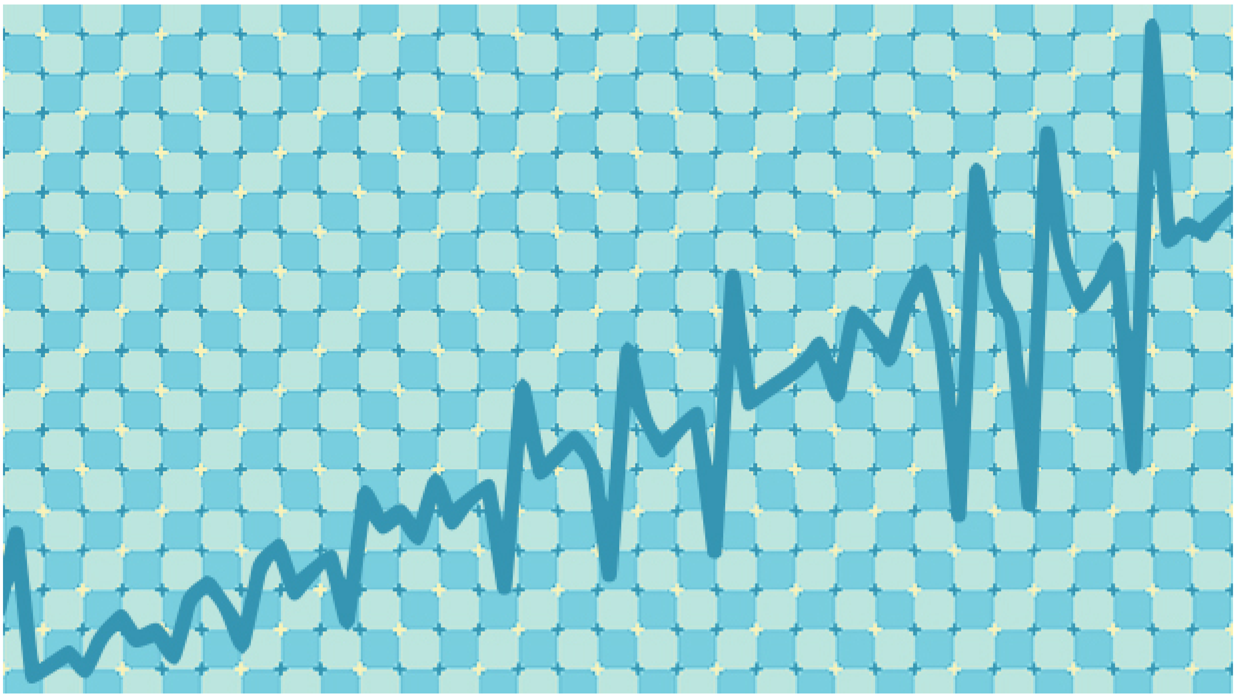
Be Still My Pulsating Sequence

Can you infer the simple rule behind a number sequence that spikes up and down like the beating of a heart?

By Pradeep Mutalik



Our Insights problem this month is based on a sequence of integers which comes from Neil Sloane, a mathematician who is arguably the world's greatest authority on such sequences. Contributing writer Erica Klarreich interviewed Sloane for *Quanta Magazine* in August, in "The Connoisseur of Number Sequences":

> Neil Sloane is considered by some to be one of the most influential mathematicians of our time.
>
> That's not because of any particular theorem the 75-year-old Welsh native has proved, though over the course of a more than 40-year research career at Bell Labs (later AT&T Labs) he won numerous awards for papers in the fields of combinatorics, coding theory, optics and statistics. Rather, it's because of the creation for which he's most famous: the Online Encyclopedia of Integer Sequences (OEIS), often simply called "Sloane" by its users.

We asked Sloane to help us construct a puzzle based on a sequence that is not in the OEIS. Here is what we came up with:

Question 1:

> Consider the sequence of numbers shown below:
>
> 13 26 2 4 6 3 9 12 8 10 5 15 18 14 7 21 24 16 …
>
> Unlike most mathematical functions, this sequence, graphed in the picture above, spikes up and down like the pulsations of the heart. Can you figure out the simple rule behind it? (**Update:** The solution is now available here.)

I love puzzles like this one because they are truly "insight" problems in the psychological sense. Most of the math we do in school involves taking a general rule — a formula or equation — and plugging in the particulars of a problem to get the solution. This process requires *deductive* reasoning, in which we go from the general to the particular. In the above question, though, you are given a particular set of numbers and you have to come up with a general rule that fits them. Here you are going in reverse — from the particular to the general. This is *inductive* reasoning. You can remember how it goes with the aid of the mnemonic PIG (particular -> inductive -> general).

InsightsPuzzle

A monthly puzzle celebrating the sudden insights and unexpected twists of scientific problem solving. Your guide is Pradeep Mutalik, a medical research scientist at the Yale Center for Medical Informatics and a lifelong puzzle enthusiast.

Problems that use inductive reasoning require a different approach from the deductive-reasoning problems we normally encounter. You have to take the particulars you are given and spend some time playing around with them, looking for patterns and trying out various candidate rules, many of which will be unsuccessful. Then suddenly, the solution may just come to you in a flash of insight — an "aha" or "eureka" moment.

If that magic moment hasn't blessed you yet and you feel that you aren't making any progress, it may help to put the problem aside and do something else for a while. A recent book titled "The Eureka Factor" by John Kounios and Mark Beeman, two of the leading researchers on this phenomenon, describes the multistage interplay between the conscious and unconscious mind that gives rise to the insight phenomenon. First you immerse your conscious mind in the problem. Often you may reach an impasse. At this point, it is best to engage in a different activity as a diversion and let your unconscious mind chew on the problem — a stage called incubation. Then, if you're patient and lucky, this underground activity can burst to the surface like a volcanic eruption and you'll have your insight and the answer. Try it and see.

If you're still not having any luck, we have a hint for you, which you can read by highlighting the blank space below (or changing the font color). Read it and try again, and may the insight strike you this time!

Once you figure out the rule behind the sequence, you are ready for the next question. (If you still can't figure it out, wait for a day, and then we will release readers' comments.)

Question 2:

> Do you think that every positive integer greater than 1 will appear in this sequence, or will some integers be skipped? Can you prove your answer?

Here's a question that may require more work:

Question 3:

> How far down the sequence do you have to go to encounter a pair of adjacent numbers that share more than one prime factor?

And finally, a tricky question:

Question 4:

> Why on earth does this sequence begin with the number 13? (Outlandish reasons are encouraged!)

To those who are chomping at the bit for more, we will discuss this sequence in more detail when we publish the solution in a couple of weeks. We'll also present an unsolved problem about it from Neil Sloane, for those intrepid readers who dream of being immortalized in the mathematical treasure trove that is the OEIS.

Happy puzzling, and may the insight strike you!

Editor's notes: The reader who submits the most interesting, creative or insightful solution (as judged by the columnist) in the comments section will receive a Quanta Magazine *T-shirt.*

If you'd like to suggest a favorite puzzle for a future Insights column, submit it as a comment below, clearly marked "NEW PUZZLE SUGGESTION" (it will not appear online, so solutions to the puzzle questions above should be submitted separately).

Note that we will hold comments for the first day to allow for independent contributions.

Update: The solution has been published here.