Safety in Numbers

A few simple calculations, along with getting vaccinated, can help prevent the spread of disease.

**INFECTION INFECTIONS**

Different infectious diseases spread through populations at different rates. A disease’s “basic reproduction number” ($R_0$) reflects how many people each infected person is expected to infect if no one has been vaccinated against the disease.

<table>
<thead>
<tr>
<th>Linear rate</th>
<th>Exponential rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>If you have a disease with an $R_0$ of 1, you are likely to infect just one new person, who could then go on to infect one more person, and so on.</td>
<td>If you have a disease with an $R_0$ of 2, you are likely to infect two new people, and each of those two will go on to infect another two. After three rounds of this, 15 people will have been infected. After 30 rounds, the number exceeds 2 billion!</td>
</tr>
</tbody>
</table>

**THE POWER OF VACCINATION**

If enough people are vaccinated against a disease, its $R_0$ can be effectively reduced to 1, ensuring that a disease will only spread at a linear rate.

Suppose you have a disease ($R_0 = 2$) and encounter 10 friends while sick ($N=10$):

- Two out of 10 friends get infected, so each friend has a 20% chance of catching your bug.
- If five friends are vaccinated ($V = 5$), those friends are very unlikely to get infected. The five unvaccinated friends still have a 20% chance of being infected, so we have effectively reduced $R_0$ to 1 (20% of 5 = 1).

A simple calculation gives us the percentage of the population that needs to be vaccinated in order to reduce $R_0$ to 1:

$$\left(1 - \frac{1}{R_0}\right) = \frac{V}{N}$$

In our example:

$$\left(1 - \frac{1}{2}\right) = \frac{5}{10}$$

**Q**

1. Suppose a disease has an $R_0$ of 4, and assume that, on average, 100 friends are encountered by an infected individual.
   A. What is the probability that any one friend becomes infected?
   B. How many of the 100 friends would have to be vaccinated to bring the effective $R$ of the disease down to 1?

2. Suppose you are infected with a seasonal flu virus ($R_0 = 1.25$).
   How many of your classmates would have to be vaccinated to reduce the effective $R_0$ of the disease to 1?

3. The percentage of vaccinations required in a population to achieve an $R_0$ of 1 is also referred to as the “herd immunity threshold” (HIT). What is the HIT of these diseases: measles ($R_0 = 12$), Ebola ($R_0 = 2.5$), smallpox ($R_0 = 5$)?

**DISCUSSION**

We claimed that after 30 rounds of infections, someone infected with a disease of $R_0 = 2$ will have infected over 2 billion people, directly or indirectly. While theoretically accurate, this is probably not true to life. What factors might prevent this?
ANSWER KEY

1. A. Four out of 100 friends will likely be infected: 4/100 = 4%
   B. Using the equation, the number that would need to be vaccinated would be: 75

\[
\left( 1 - \frac{1}{R_0} \right) = \frac{V}{N} \quad \left( 1 - \frac{1}{4} \right) = \frac{V}{100} \quad \left( 1 - \frac{1}{4} \right) = \frac{75}{100}
\]

2. This answer will depend on the number of students in your class.
   If you happen to have 25 classmates, we see that five would need to be vaccinated:

\[
\left( 1 - \frac{1}{R_0} \right) = \frac{V}{N} \quad \left( 1 - \frac{1}{1.25} \right) = \frac{V}{25} \quad \left( 1 - \frac{1}{1.25} \right) = \frac{5}{25}
\]

Notice that diseases with lower reproduction numbers require fewer people to be vaccinated in order to reduce \( R_0 \) to 1.

3. Using the equation:

\[
\left( 1 - \frac{1}{R_0} \right) = \text{HIT}
\]

Measles: \( \left( 1 - \frac{1}{12} \right) = 0.917 \) (91.7%)
Ebola: \( \left( 1 - \frac{1}{2.5} \right) = 0.6 \) (60%)
Smallpox: \( \left( 1 - \frac{1}{5} \right) = 0.8 \) (80%)

DISCUSSION NOTES

- Some infected people are likely to stay home, reducing their contact with others.
- As the infection spreads and more people get sick, there are fewer people left to infect.
- Social relationships generally aren’t independent. Suppose Allen infects Betty and Chris.
  We know that Betty and Chris already share one acquaintance (Allen), which means they probably share others as well. They may end up infecting the same people, reducing the overall rate of infection.